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Parameter identification for a nonlinear non-smooth thermodynamic system of sea ice

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Abstract

This study is intended to provide a method to estimate the sea ice parameters in a nonlinear non-smooth thermodynamic system (NNTS) from the sea ice temperature measurements. A new optimization algorithm is constructed to estimate the coefficients describing the salinity and other two parameters in NNTS. Another simulation for sea ice temperature during different measurement period is operated. Results show that a better simulation of the temperature distribution is possible with the estimated parameters than with the experimental parameters.

Keywords: Parameter identification; Nonlinear non-smooth thermodynamic system; Optimization algorithm; Sea ice

1. Introduction

Sea ice plays an important role in the global climate system [1]. Its freezing and melting processes are influenced by the temperature distribution, thus the numerical simulation for the sea ice temperature distribution have attracted great attention [2–8]. However, the physical parameters such as the density, the specific heat, the thermal conductivity and the salinity, and so on, are crucial for exactly describing the sea ice temperature profile. Therefore, accurately estimating these physical parameters can improve the sea ice thermodynamic modeling.

Until now, these physical parameters in the sea ice thermodynamic system are mainly estimated by field data [3,4]. However, the field data are spare and unsatisfactory due to the difficulties associated with fieldwork, especially during the polar winter. Some parameters could not be detected continuously and automatically up to now, such as the ice salinity; some could not be detected, such as the ice thermal conductivity. Thus it could not help us to thoroughly understand the physical evolution of sea ice just by field data. The parameter identification method is effective to solve this problem. Parameter identification refers to the determination of the physical parameters that could be detected discontinuously or difficultly from the physical parameters which can be detected continuously in the system model such that the predicted response of the model is close, in some well-defined sense, to the process observations. The parameter identification method can reinforce the field data for improved understanding of the salinity evolution of sea ice because sea ice temperature can be measured continuously and automatically while sea ice salinity only can be measured aperiodically. In recent years, there are many researchers devoting to the parameter identification problems of thermodynamic systems. Some researchers [9–11] considered the determination of source terms, some researchers [12–14] considered the determination of thermal conductivities. In [15], Lv, Feng and Li identified the densities, the specific heats and the thermal conductivities of a coupled thermodynamic system of sea ice. In this paper, we will deal with a nonlinear and non-smooth thermodynamic system (NNTS) of sea ice, and identify the coefficients describing the sea ice salinity and other two parameters in NNTS using the parameter identification method. Our method allows one to estimate the sea-ice salinity distribution from temperature measurements alone. It might, in conjunction with measurements, guide the way towards an improved understanding of the salinity evolution of sea ice, which is crucial

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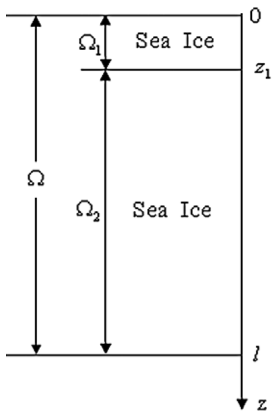

Fig. 1. The configuration of the thermodynamic model of sea ice.

to describe sea ice in large scale climate models, for example. However, currently the impact of snow on sea ice, of the oceanic heat flux and of radiation penetrating through snow on top of the sea ice is not included in our mathematical framework. Additionally, we do not account for the impact of brine moving within sea ice on the temperature field. To overcome these limitations will be subject to future work. Applying our model with obtained optimal parameters to field data shows that despite these limitations, a better simulation of the observed temperature distribution is possible with our model than would be using a standard salinity profile. Therefore, our method can help in interpreting field data and can be used, for example, to overcome data gaps.

The rest of this paper is organized as follows. In Section 2, we present a nonlinear non-smooth thermodynamic system of sea ice, and the existence and uniqueness of weak solution of NNTS. In Section 3, a parameter identification model is established, the dependency relationship between the state and control variables is proved, and the existence of optimal control is presented; moreover, we construct an optimization algorithm to find the optimal solution and make another numerical simulation during the Polar Night Time. In Section 4, we conclude this research and present the forthcoming work.

2. The nonlinear non-smooth thermodynamic system of sea ice

Since the gradient variation of sea ice temperature in the vertical direction is far greater than that in the horizontal direction, we only consider the heat flux in the vertical direction. The sea ice system (SIS) is shown in Fig. 1.

In Fig. 1, the vertical coordinate z representing the depth of the system is taken as positive downward, take some point at the sea ice surface as the origin; let l be the total depths of SIS, z_1 be the depth of sea ice between 0 and l determined by the source term; ice depth is in meters. Let t denote time, t_f (>0) denote the final time, time is in seconds. Let $T = T(z,t)$ be the temperature at depth z and time t, $T_0(z)$ be the initial temperature; $T_U = T_U(t)$, $T_L = T_L(t)$ and $T_{z_1} = T_{z_1}(t)$ denote the temperature functions at $z = 0$, $z = l$ and $z = z_1$, respectively; ice temperature is in Kelvins. Let $\Omega_1 = (0, z_1)$, $\Omega_2 = (z_1, l)$, $\Omega = \Omega_1 \cup \{z_1\} \cup \Omega_2 = (0, l) \subset R$, $I = (0, t_f)$, $Q = \Omega \times I$, $Q_i = \Omega_i \times I$, $i \in I_2$, where $I_2 = \{1, 2\}$. Let $h(t)$ be the thickness function of sea ice on t, $\hat{z}$ be the normalized-depth, that is $\hat{z} = z/h(t)$, and $h(t) = h_1 + h_2 t > 0$, h_1 and h_2 are given positive constants, $(z,t) \in \overline{Q}$. For convenience, set $I_n = \{1, 2, \ldots, n\}$, n is any positive integer. According to the Fourier's law of heat conduction, the thermodynamic system of SIS is described by the following heat conduction equations denoted by NNTS.

$$(\rho \cdot c)(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right) + F(z), \quad (z,t) \in Q, \tag{1}$$

$$T(z,0) = T_0(z), \quad z \in \Omega, \tag{2}$$

$$T(0,t) = T_U(t), \quad t \in \bar{I}, \tag{3}$$

$$T(l,t) = T_L(t), \quad t \in \bar{I}. \tag{4}$$

The product formulae of the density and the specific heat $(\rho \cdot c)(T)$ of sea ice and the thermal conductivity $k(T)$ are expressed by the following equations on the basis of previous work [2–4,7].

$$(\rho \cdot c)(T) = \rho_{\mathrm{pi}} c_{\mathrm{pi}} + \frac{\gamma S(\hat{z})}{(T - 273.15)^2}, \quad (z,t) \in Q, \tag{5}$$

$$k(T) = k_{\mathrm{pi}} + \frac{\beta S(\hat{z})}{T - 273.15}, \quad (z,t) \in Q, \tag{6}$$

$$S(\hat{z}) = a_0 + a_1 \hat{z} + a_2 \hat{z}^2 + a_3 \hat{z}^3, \quad (z,t) \in Q, \tag{7}$$

$$F(z) = \begin{cases} \kappa_1 (1 - \bar{\alpha}) Q_s e^{-\kappa_1 z}, & 0 \leqslant z < z_1, \\ i_0 \kappa_2 (1 - \bar{\alpha}) Q_s e^{-\kappa_2 (z - z_1)}, & z_1 \leqslant z \leqslant l. \end{cases} \tag{8}$$

Where ρ_{pi}, c_{pi} and k_{pi} are the density, the specific heat and the thermal conductivity of pure ice, respectively; γ and β are positive parameters. $S(\hat{z})$ is the sea ice salinity at normalized-depth $\hat{z}$, and it is in parts per thousand; a_0, a_1, a_2 and a_3 are parameters corresponding to the normalized-depth. $F(z)$ is the source term; i_0 is the fraction of the net absorbed solar radiation penetrating into the interior of the ice to form brine pockets when the surface is snow-free, and $i_0 = 0.18(1 - C) + 0.35C$, where C is a sky condition constant between 0 and 1; $\bar{\alpha}$ is the sea ice albedo; Q_s is the flux of solar shortwave radiation; κ_1 and κ_2 are the bulk shortwave extinction coefficients of sea ice, and $\kappa_1 = -10 \times \ln i_0$.

Since the density, the specific heat and the thermal conductivity of sea ice are relevant to the sea ice temperature, NNTS is nonlinear; since the source term is a piecewise function, NNTS is non-smooth. From the above analysis, we can get that NNTS is a nonlinear non-smooth system.

According to the physical characteristics of SIS, we make the following assumptions:

- (A1) *The temperature $T(z,t)$ of NNTS is continuous on $\overline{Q}$, and there exist $T_{\inf} > 1$, $0 < T_{\sup} < 273$, and $T_{\inf} \leqslant T_{\sup}$, such that $T_{\inf} \leqslant T(z,t) \leqslant T_{\sup}$, for any $(z,t) \in \overline{Q}$. And there exists $L > 0$, for any $(z,t) \in \overline{Q}$, such that $|\partial T/\partial z| \leqslant L$.*
- (A2a) *Let $q = (q_1, q_2, q_3, q_4, q_5, q_6) = (\gamma, \beta, a_0, a_1, a_2, a_3)$, $\gamma, \beta, a_0, a_1, a_2$ and a_3 are bounded, that is, there exist $q_{Lj} \in R$, $q_{Uj} \in R$, and $q_{Lj} \leqslant q_{Uj}$ ($j \in I_6$), such*

that $q_{L1} \leqslant \gamma \leqslant q_{U1}$, $q_{L2} \leqslant \beta \leqslant q_{U2}$, $q_{L3} \leqslant a_0 \leqslant q_{U3}$, $q_{L4} \leqslant a_1 \leqslant q_{U4}$, $q_{L5} \leqslant a_2 \leqslant q_{U5}$, $q_{L6} \leqslant a_3 \leqslant q_{U6}$.

From assumption (A2a), let $Q_{\mathrm{ad}} = \{q = (q_1, q_2, q_3, q_4, q_5, q_6) \mid q_{Lj} \leqslant q_j \leqslant q_{Uj}, j \in I_6$, and $S_1 \leqslant q_3 + q_4\hat{z} + q_5\hat{z}^2 + q_6\hat{z}^3 \leqslant S_2$, $S_3 \leqslant q_3 + q_4 + q_5 + q_6 \leqslant S_4$, where $\hat{z} = z/h(t)$, $\forall(z,t) \in \overline{Q}\}$ be a parameter set, then Q_{ad} is a compact convex subset of $\mathbb{R}^6$, where S_1 and S_2 are given positive constants. To illustrate the relationship between the density, the specific heat, the thermal conductivity of sea ice and the parameter $q \in Q_{\mathrm{ad}}$, rewrite $(\rho\cdot c)(T)$ and $k(T)$ as $(\rho\cdot c)(T,q)$ and $k(T,q)$, respectively. Let $z_0 = 0$, $T_U(t) = T_{z_0}(t)$, $z_2 = l$, $T_L(t) = T_{z_2}(t)$.

(A2b) *There exist* $M_m > 0$ *and* $L_n > 0$, $m \in I_2$, $n \in I_4$, *such that*

$$\left|\frac{\partial(\rho\cdot c)(T,q)}{\partial T}\right| \leqslant M_1, \qquad \left|\frac{\partial k(T,q)}{\partial T}\right| \leqslant M_2, \quad \forall T \in C(\overline{Q};\mathbb{R}),\ \forall q \in Q_{\mathrm{ad}}$$

and

$$\left|(\rho\cdot c)(T,q') - (\rho\cdot c)(T,q'')\right| \leqslant L_1\|q'-q''\|_{L^2(Q_{\mathrm{ad}})},$$
$$\left|k(T,q') - k(T,q'')\right| \leqslant L_2\|q'-q''\|_{L^2(Q_{\mathrm{ad}})},$$
$$\left|\frac{\partial(\rho\cdot c)(T,q')}{\partial T} - \frac{\partial(\rho\cdot c)(T,q'')}{\partial T}\right| \leqslant L_3\|q'-q''\|_{L^2(Q_{\mathrm{ad}})},$$
$$\left|\frac{\partial k(T,q')}{\partial T} - \frac{\partial k(T,q'')}{\partial T}\right| \leqslant L_4\|q'-q''\|_{L^2(Q_{\mathrm{ad}})}, \quad \forall T \in C(\overline{Q};\mathbb{R}),\ \forall q', q'' \in Q_{\mathrm{ad}}.$$

(A3) $S(\hat{z}) \in C^1(Q)$, $h(t) \in C^2(I;R^+)$, $F(z) \in C^1(\Omega_i)$, *and there exists* $\overline{L} > 0$, *for any* $z \in \overline{\Omega}_i$, *such that* $|F(z)| \leqslant \overline{L}$, $i \in I_2$; $T_{z_0}(t)$, $T_{z_1}(t)$, $T_{z_2}(t) \in C^1(I)$, *and there exists* $M_3 > 0$, *such that* $0 < T_{z_i}(t) - T_{z_{i-1}}(t) \leqslant M_3$, $\forall t \in I$, $i \in I_2$.

(A4) $T_0(z) \in H_0^1(\Omega_i) \cap H^2(\Omega_i)$, *and there exists* $\varepsilon_0 > 0$, *for any* $(z,t) \in Q_i$, *such that* $|\partial^2 T_0/\partial z^2| < \varepsilon_0$, $i \in I_2$; $T_U(0) = T_0(0)$, $T_L(0) = T_0(l)$.

Since $T_{z_1}(t)$, $T_0(z)$, $T_U(t)$ and $T_L(t)$ are all given, the temperature $T(z,t)$ of NNTS is dependent on the parameters $\gamma, \beta, a_0, a_1, a_2$ and a_3, that is, $T(z,t) = T(z,t;\gamma,\beta,a_0,a_1,a_2,a_3)$. Thus $\gamma, \beta, a_0, a_1, a_2$ and a_3 are to be identified later.

Remark 1. Let

$$(\rho c)_L = \rho_{\mathrm{pi}}c_{\mathrm{pi}} + \min_{i\in I_2}\left\{\inf_{(z,t)\in Q_i}\frac{q_1 S(\hat{z})}{(T-273.15)^2}\right\},$$
$$(\rho c)_U = \rho_{\mathrm{pi}}c_{\mathrm{pi}} + \max_{i\in I_2}\left\{\sup_{(z,t)\in Q_i}\frac{q_1 S(\hat{z})}{(T-273.15)^2}\right\},$$
$$k_L = k_{\mathrm{pi}} + \min_{i\in I_2}\left\{\inf_{(z,t)\in Q_i}\frac{q_2 S(\hat{z})}{T-273.15}\right\},$$
$$k_U = k_{\mathrm{pi}} + \max_{i\in I_2}\left\{\sup_{(z,t)\in Q_i}\frac{q_2 S(\hat{z})}{T-273.15}\right\},$$

from assumptions (A1)–(A3), we have that

$$(\rho c)_L \leqslant (\rho\cdot c)(T,q) \leqslant (\rho c)_U, \quad \forall T \in C(\overline{Q};\mathbb{R}),\ \forall q \in Q_{\mathrm{ad}}$$
$$k_L \leqslant k(T,q) \leqslant k_U, \quad \forall T \in C(\overline{Q};\mathbb{R}),\ \forall q \in Q_{\mathrm{ad}}.$$

From the above analysis, we decompose NNTS into two subsystems.

Subsystem (1):

$$\begin{aligned}
&(\rho\cdot c)(T,q)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k(T,q)\frac{\partial T}{\partial z}\right) + F(z), && (z,t)\in Q_1,\\
&T(z,0) = T_0(z), && z \in \Omega_1,\\
&T(z_0,t) = T_{z_0}(t), && t \in \bar{I},\\
&T(z_1,t) = T_{z_1}(t), && t \in \bar{I},
\end{aligned}$$

where

$$\begin{aligned}
&(\rho\cdot c)(T,q) = \rho_{\mathrm{pi}}c_{\mathrm{pi}} + \frac{\gamma S(\hat{z})}{(T-273.15)^2}, && (z,t)\in Q_1,\\
&k(T,q) = k_{\mathrm{pi}} + \frac{\beta S(\hat{z})}{T-273.15}, && (z,t)\in Q_1,\\
&S(\hat{z}) = a_0 + a_1\hat{z} + a_2\hat{z}^2 + a_3\hat{z}^3, && (z,t)\in Q_1,\\
&F(z) = \kappa_1(1-\bar{\alpha})Q_s e^{-\kappa_1 z}, && z \in \Omega_1.
\end{aligned}$$

Subsystem (2):

$$\begin{aligned}
&(\rho\cdot c)(T,q)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k(T,q)\frac{\partial T}{\partial z}\right) + F(z), && (z,t)\in Q_2,\\
&T(z,0) = T_0(z), && z \in \Omega_2,\\
&T(z_1,t) = T_{z_1}(t), && t \in \bar{I},\\
&T(z_2,t) = T_{z_2}(t), && t \in \bar{I},
\end{aligned}$$

where

$$\begin{aligned}
&(\rho\cdot c)(T,q) = \rho_{\mathrm{pi}}c_{\mathrm{pi}} + \frac{\gamma S(\hat{z})}{(T-273.15)^2}, && (z,t)\in Q_2,\\
&k(T,q) = k_{\mathrm{pi}} + \frac{\beta S(\hat{z})}{T-273.15}, && (z,t)\in Q_2,\\
&S(\hat{z}) = a_0 + a_1\hat{z} + a_2\hat{z}^2 + a_3\hat{z}^3, && (z,t)\in Q_2,\\
&F(z) = i_0\kappa_2(1-\bar{\alpha})Q_s e^{-\kappa_2(z-z_1)}, && z \in \Omega_2.
\end{aligned}$$

And $T_{\Omega_1}(z_1,t) = T_{\Omega_2}(z_1,t)$, $\frac{\partial T_{\Omega_1}}{\partial \vec{n}_1}|_{z=z_1} = \frac{\partial T_{\Omega_2}}{\partial \vec{n}_2}|_{z=z_1}$, where $T_{\Omega_i,t}$ and $\frac{\partial T_{\Omega_i}}{\partial \vec{n}_i}$ are the temperature in Ω_i and normal derivatives at z_1, $i \in I_2$, for any $t \in \bar{I}$.

For the subsystem (i), for any $(z,t) \in \overline{Q}_i$, $i \in I_2$, let $u = u(z,t) = T(z,t) - (1 - \frac{z-z_{i-1}}{z_i-z_{i-1}})T_{z_{i-1}}(t) - \frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)$, then the subsystem (i) $(i \in I_2)$ can be rewritten as

$$(\rho\cdot c)(u,q)\frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left(k(u,q)\frac{\partial u}{\partial z}\right) + \frac{T_{z_i}(t) - T_{z_{i-1}}(t)}{z_i - z_{i-1}}\frac{\partial k(u,q)}{\partial z} + \overline{F}(u,q), \quad (z,t)\in Q_i, \tag{9}$$

$$u(z,0)=u_0(z), \quad z\in\Omega_i, \tag{10}$$

$$u(z_{i-1},t)=0, \quad t\in\bar I, \tag{11}$$

$$u(z_i,t)=0, \quad t\in\bar I, \tag{12}$$

where

$$(\rho\cdot c)(u,q)=\rho_{\mathrm{pi}}c_{\mathrm{pi}}+\gamma S(\hat z)\Big/\left(u+\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)-273.15\right)^2, \quad (z,t)\in Q_i, \tag{13}$$

$$k(u,q)=k_{\mathrm{pi}}+\beta S(\hat z)\Big/\left(u+\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)-273.15\right), \quad (z,t)\in Q_i, \tag{14}$$

$$\overline F(u,q)=F(z)-(\rho\cdot c)(u,q)\left(\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T'_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T'_{z_i}(t)\right), \quad (z,t)\in Q_i, \tag{15}$$

$$u_0(z)=T_0(z)-\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(0)-\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(0), \quad z\in\Omega_i. \tag{16}$$

For convenience of our subsequent analysis, we use the following notation: $u_t=\frac{\partial u}{\partial t}$, $\nabla u=\frac{\partial u}{\partial z}$, $\triangle u=\nabla^2 u$. Since $(\rho\cdot c)(u;q)>0$, dividing (9) by $(\rho\cdot c)(u;q)$, then the subsystem (i) ($i\in I_2$) can be transformed into the following format denoted by NNTSS:

$$u_t-\nabla\big(a(u,q)\nabla u\big)-b(u,q)|\nabla u|^2+e(u,q)\nabla u=f(u,q), \quad (z,t)\in Q_i, \tag{17}$$

$$u(z,t)=0, \quad (z,t)\in\big(\{z_{i-1}\}\cup\{z_i\}\big)\times\bar I, \tag{18}$$

$$u(z,0)=u_0(z), \quad z\in\Omega_i. \tag{19}$$

Where

$$a(u,q)=\frac{k(u,q)}{(\rho\cdot c)(u,q)}, \tag{20}$$

$$b(u,q)=-2\gamma S(\hat z)k(u,q)\big/(\rho\cdot c)^2(u,q)\times\left(u+\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)-273.15\right)^3, \tag{21}$$

$$\begin{aligned}e(u,q)={}&-\gamma k(u,q)\frac{\partial S(\hat z)}{\partial z}\big/(\rho\cdot c)^2(u,q)\times\left(u+\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)-273.15\right)^2\\&-2\gamma S(\hat z)k(u,q)\big/(\rho\cdot c)^2(u,q)\times\left(u+\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)-273.15\right)^3\cdot\frac{T_{z_i}(t)-T_{z_{i-1}}(t)}{z_i-z_{i-1}}\\&-\beta S(\hat z)/(\rho\cdot c)(u,q)\left(u+\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)-273.15\right)^2\cdot\frac{T_{z_i}(t)-T_{z_{i-1}}(t)}{z_i-z_{i-1}},\end{aligned} \tag{22}$$

$$\begin{aligned}f(u,q)={}&\beta\frac{\partial S(\hat z)}{\partial z}/(\rho\cdot c)(u,q)\left(u+\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)-273.15\right)\cdot\frac{T_{z_i}(t)-T_{z_{i-1}}(t)}{z_i-z_{i-1}}\\&-\beta S(\hat z)/(\rho\cdot c)(u,q)\left(u+\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T_{z_i}(t)-273.15\right)^2\\&\times\left(\frac{T_{z_i}(t)-T_{z_{i-1}}(t)}{z_i-z_{i-1}}\right)^2+\frac{F(z)}{(\rho\cdot c)(u,q)}\\&-\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T'_{z_{i-1}}(t)-\frac{z-z_{i-1}}{z_i-z_{i-1}}T'_{z_i}(t).\end{aligned} \tag{23}$$

Remark 2. From assumptions (A1), (A3) and the definition of $u(z,t)$, we can conclude that $u(z,t)$ is continuous on Q_i, $i\in I_2$; from (13), (14), the definition of $u(z,t)$ and Remark 1, for any $u\in C(Q_i;\mathbb R)$, any $q\in Q_{\mathrm{ad}}$, $i\in I_2$, we have that $(\rho c)_L\leqslant(\rho\cdot c)(u,q)\leqslant(\rho c)_U$, $k_L\leqslant k(u,q)\leqslant k_U$; from assumptions (A1)–(A3), (15) and Remark 1, we can obtain that $\overline F(u,q)\in C(C(Q_i;R)\times Q_{\mathrm{ad}};\mathbb R)$, $i\in I_2$; from assumptions (A1)–(A3), for any $q',q''\in Q_{\mathrm{ad}}$, $u\in C(Q_i;R)$, $i\in I_2$, take

$$L_5=L_1\max_{i\in I_2}\left\{\sup_{(z,t)\in Q_i}\left\{\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T'_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T'_{z_i}(t)\right\}\right\},$$

we can get that

$$\left|\frac{\partial(\rho\cdot c)(u,q')}{\partial u}-\frac{\partial(\rho\cdot c)(u,q'')}{\partial u}\right|\leqslant L_3\|q'-q''\|_{L^2(Q_{\mathrm{ad}})},$$

$$\left|\frac{\partial k(u,q')}{\partial u}-\frac{\partial k(u,q'')}{\partial u}\right|\leqslant L_4\|q'-q''\|_{L^2(Q_{\mathrm{ad}})},$$

$$\left|\overline F(u,q')-\overline F(u,q'')\right|\leqslant L_5\|q'-q''\|_{L^2(Q_{\mathrm{ad}})},$$

and for any $q\in Q_{\mathrm{ad}}$, any $u\in C(Q_i;\mathbb R)$, any $(z,t)\in Q_i$, $i\in I_2$, take

$$M_4=(\rho c)_U\max_{i\in I_2}\left\{\sup_{(z,t)\in Q_i}\left\{\left(1-\frac{z-z_{i-1}}{z_i-z_{i-1}}\right)T'_{z_{i-1}}(t)+\frac{z-z_{i-1}}{z_i-z_{i-1}}T'_{z_i}(t)\right\}\right\},$$

$M_5 = 2T_{\sup} + M_3,$

$M_6 = L + M_3 \max\left\{\frac{1}{z_1}, \frac{1}{l - z_1}\right\},$

we can obtain that

$$\left|\frac{\partial(\bar{\rho}\cdot\bar{c})(u,q)}{\partial u}\right| \leqslant M_1, \qquad \left|\frac{\partial\bar{k}(u,q)}{\partial u}\right| \leqslant M_2,$$

$$\left|\overline{F}(u,q)\right| \leqslant M_4, \qquad \left|u(z,t)\right| \leqslant M_5, \qquad \left|\nabla u(z,t)\right| \leqslant M_6.$$

Remark 3. From assumptions (A1)–(A4), (20)–(23), we can get that $a(u,q)$, $b(u,q)$, $e(u,q)$, $f(u,q) \in C^1(C(Q_i;\mathbb{R}) \times Q_{\mathrm{ad}};\mathbb{R})$, $i \in I_2$, and there exist positive constants $\tilde{a}_0$, $\tilde{a}_1$, and $\tilde{a}_0 \leqslant \tilde{a}_1$, such that $\tilde{a}_0 \leqslant a(u,q) \leqslant \tilde{a}_1$; take $M_7 = M_4(\rho c)_L^{-1}$, and we have $|f(u,q)| \leqslant M_7$, for any $q \in Q_{\mathrm{ad}}$, any $u \in C(Q_i;\mathbb{R})$, $i \in I_2$.

Remark 4. From assumptions (A1)–(A4), (20)–(23) and Remark 2, for any $u \in C(Q_i;\mathbb{R})$, $i \in I_2$, take

$$\widetilde{M} = \sup_{s\in C(Q_i;\mathbb{R})}\left\{\frac{\partial a}{\partial u}(s), \frac{\partial b}{\partial u}(s)\right\},$$

$$\overline{M} = \sup_{s\in C(Q_i;\mathbb{R})}\left\{\frac{\partial e}{\partial u}(s), \frac{\partial f}{\partial u}(s)\right\},$$

we can obtain that

$$\max\left\{\left\|\frac{\partial a}{\partial u}\right\|_{C(C(Q_i;R)\times Q_{\mathrm{ad}};\mathbb{R})}, \left\|\frac{\partial b}{\partial u}\right\|_{C(C(Q_i;R)\times Q_{\mathrm{ad}};\mathbb{R})}\right\} \leqslant \widetilde{M},$$

$$\max\left\{\left\|\frac{\partial e}{\partial u}\right\|_{C(C(Q_i;R)\times Q_{\mathrm{ad}};\mathbb{R})}, \left\|\frac{\partial f}{\partial u}\right\|_{C(C(Q_i;R)\times Q_{\mathrm{ad}};\mathbb{R})}\right\} \leqslant \overline{M}|u|.$$

Take $M = \max\{\widetilde{M}, \overline{M}\}$, therefore, for any $q \in Q_{\mathrm{ad}}$, such that $|b(u,q)| \leqslant M|u|$, $|e(u,q)| \leqslant M|u|^2$, $|f(u,q)| \leqslant M|u|^2$.

Remark 5. From assumption (A4) and (16), we can obtain that $u_0 \in H_0^1(\Omega_i) \cap H^2(\Omega_i)$, and $\|\triangle u_0(z)\|_{L^2(\Omega_i)} < \varepsilon_0$, for some $\varepsilon_0 > 0$, for any $z \in \Omega_i$, $i \in I_2$.

From the above analysis, we can get the following results.

Theorem 1. *Suppose assumptions* (A1)–(A4) *hold, then NNTSS has a unique solution* $u(z,t): Q_i \to \mathbb{R}$, $i \in I_2$.

Proof. The proof is as in Ref. [16]. □

Theorem 2. *Suppose assumptions* (A1)–(A4) *hold, then NNTS has a unique weak solution* $T(z,t): Q \to R$.

Proof. According to Ref. [17] and Theorem 1, we can obtain that NNTS has a unique weak solution $T(z,t): Q \to \mathbb{R}$. □

Define S be the set of weak solutions of NNTS, i.e. $S := \{T(z,t;q) \mid T(z,t;q)$ is the weak solution of NNTS corresponding to $q \in Q_{\mathrm{ad}}\}$.

3. Parameter identification for the nonlinear non-smooth thermodynamic system

In this section, we will consider the parameter identification problem of NNTS. Let $\widetilde{T}_{z,t}$ be the observed temperature data of NNTS at depth z and time t, $\overline{T}(z,t)$ be the fitted continuous temperature function of $\widetilde{T}_{z,t}$, where $(z,t) \in \overline{Q}$. Define the performance criterion

$$J(q) = \left\|T(z,t;q) - \overline{T}(z,t)\right\|^2_{L^2(Q)}, \tag{24}$$

where $T(z,t;q) \in S(Q_{\mathrm{ad}})$. Our goal is to make the temperature $T(z,t;q)$ of NNTS approach the fitted function $\overline{T}(z,t)$, so the optimal control model can be expressed as

$$\text{(PIP)} \quad \min J(q) \quad \text{s.t. } T(z,t;q) \in S(Q_{\mathrm{ad}}), q \in Q_{\mathrm{ad}}. \tag{25}$$

3.1. Existence of optimal parameter

First we will consider the existence of optimal control. We will give a preparative theorem.

Theorem 3. *Suppose assumptions* (A1)–(A4) *hold, then the mapping* $q \to T(z,t;q)$ *is strongly continuous, for any* $q \in Q_{\mathrm{ad}}$.

Proof. We first prove that the mapping $q \to u(t;q)$ is strongly continuous, where $u(q)$ is the weak solution of NNTSS, for any $q \in Q_{\mathrm{ad}}$. For any fixed parameter $q_0 = (\gamma_0, \beta_0, a_0^0, a_1^0, a_2^0, a_3^0) \in Q_{\mathrm{ad}}$, let $\{q_n\} \subset Q_{\mathrm{ad}}$ be a sequence, such that $\|q_n - q_0\|_{L^2(Q_{\mathrm{ad}})} \to 0$, as $n \to \infty$, where $q_n = (\gamma_n, \beta_n, a_0^n, a_1^n, a_2^n, a_3^n)$; let $u_n = u_n(t;q_n)$ and $u^0 = u^0(t;q_0)$ be the weak solutions of NNTSS corresponding to q_n and q_0, respectively; set $\omega_n = \omega_n(z,t) = u_n - u^0$, $\bar{a}_n = a(u_n,q_n)$, $\bar{a}_0 = a(u^0,q_0)$, $b_n = b(u_n,q_n)$, $b_0 = b(u^0,q_0)$, $e_n = e(u_n,q_n)$, $e_0 = e(u^0,q_0)$, $f_n = f(u_n,q_n)$, $f_0 = f(u^0,q_0)$, then for $i \in I_2$, we have

$$\begin{aligned}
&\int_{\Omega_i} \omega_n\omega_n'\,dz - \int_{\Omega_i} \omega_n\nabla(\bar{a}_n\nabla\omega_n)\,dz \\
&\quad - \int_{\Omega_i} \omega_n\nabla\big((\bar{a}_n - \bar{a}_0)\nabla u^0\big)\,dz \\
&\quad - \int_{\Omega_i} \omega_n b_n\nabla\omega_n\nabla\big(u_n + u^0\big)\,dz + \int_{\Omega_i} \omega_n(b_0 - b_n)|\nabla u|^2\,dz \\
&\quad + \int_{\Omega_i} \omega_n e_n\nabla\omega_n\,dz + \int_{\Omega_i} \omega_n(e_n - e_0)\nabla u\,dz \\
&= \int_{\Omega_i} \omega_n(f_n - f_0)\,dz, \quad (z,t) \in Q_i,
\end{aligned} \tag{26}$$

$$\omega_n(z,t) = 0, \quad (z,t) \in \big(\{z_{i-1}\} \cup \{z_i\}\big) \times \bar{I}, \tag{27}$$

$$\omega_n(z,0) = 0, \quad z \in \Omega_i. \tag{28}$$

Integrate (26) over $(0,t)$, we can obtain that

$$\|w_n(t)\|^2_{L^2(\Omega_i)} + \int_0^t \|w_n(s)\|^2_{H^1_0(\Omega_i)} ds \leqslant \frac{Y_1}{\hat{a}_0}\|q_n - q\|^2_{L^2(Q_{ad})} + \frac{Y_2}{\hat{a}_0}\int_0^t \|w_n(s)\|^2_{L^2(\Omega_i)} ds,$$

where

$$\hat{a}_0 = \min\left\{\frac{1}{2}, \frac{\bar{a}_0}{4}\right\},$$
$$y_1 = M_6(\rho c)_L^{-2}\big(k_U L_1 + (\rho c)_U L_2\big),$$
$$y_2 = M_3 M_6 l^{-1}(\rho c)_L^{-1}\big(L_1 M_2 (\rho c)_L^{-1} + L_4\big) + (\rho c)_L^{-2}\big(L_1 M_4 + L_5(\rho c)_U\big),$$
$$y_3 = M_6^2(\rho c)_L^{-2}\big(2M_1 L_1 (\rho c)_L^{-2}(\rho c)_U k_U + M_1 L_2 (\rho c)_L^{-2}(\rho c)_U^2 + L_3 k_U\big),$$
$$Y_1 = 2a_0^{-1} t_f\big(y_1^2 + (y_2^2 + y_3^2)^2\big),$$
$$Y_2 = a_0^{-1} M M_5 C_0^2(2M_6 + 1).$$

Using the Gronwall inequality, we have that

$$\|w_n(t)\|^2_{L^2(\Omega_i)} + \int_0^t \|w_n(s)\|^2_{H^1_0(\Omega_i)} ds \leqslant \frac{Y_1}{\hat{a}_0}\|q_n - q_0\|^2_{L^2(Q_{ad})} e^{\hat{a}_0^{-1} Y_2 t_f}. \tag{29}$$

For (29), let $n \to \infty$, since $\|q_n - q_0\|_{L^2(Q_{ad})} \to 0$, $u_n \to u$ on Q_i, $i \in I_2$. We conclude that the mapping $q \to u(z,t;q)$ is strongly continuous on Q_{ad}, for $(z,t) \in Q_i$, $i \in I_2$. Thus the mapping $q \to T(z,t;q)$ is strongly continuous on Q_{ad}, where $T(z,t;q) \in S(Q_{ad})$, $q \in Q_{ad}$, which completes the proof. □

Theorem 4. *Suppose assumptions* (A1)–(A4) *hold, then there exists at least one optimal parameter* $q^* \in Q_{ad}$ *satisfying* (PIP).

Proof. Since Q_{ad} is a nonempty convex subset of R^6, there exists a minimizing sequence $\{q_n\}$, such that, $\lim_{n\to\infty} J(q_n) = \inf_{q\in Q_{ad}} J(q)$. Since Q_{ad} is compact, there exists a subsequence $\{q_{n_{\bar{k}}}\}$ and a $q^* \in Q_{ad}$, such that, $\|q_{n_{\bar{k}}} - q^*\|_{L^2(Q_{ad})} \to 0$. From Theorem 3, $q \to T(z,t;q)$ is continuous for all $q \in Q_{ad}$, $T(x,t;q_{n_{\bar{k}}}) \to T(z,t;q^*)$. It follows from (24) that

$$J(q^*) = J\Big(\lim_{\bar{k}\to\infty} q_{n_{\bar{k}}}\Big) = \lim_{k\to\infty} J(q_{n_{\bar{k}}}) = \inf_{q\in Q_{ad}} J(q).$$

This proves that q^* is an optimal parameter. □

3.2. Optimization algorithm

In this section, we will construct a feasible algorithm to solve the problem (PIP). First (25) can be written as the equivalent formula

$$\min J(q)$$
$$\text{s.t.}\quad T(z,t;q) \in S(Q_{ad}),$$
$$S_1 \leqslant q_3 + q_4\hat{z} + q_5\hat{z}^2 + q_6\hat{z}^3 \leqslant S_2, \quad \forall z \in \overline{\Omega},$$
$$S_3 \leqslant q_3 + q_4 + q_5 + q_6 \leqslant S_4,$$
$$q_{lj} \leqslant q_j \leqslant q_{uj}, \quad j \in I_6. \tag{30}$$

Where S_i are given positive constants, $i \in I_4$ which are determined by the physical properties of sea ice in the measured fields.

Let j_z be the observed spatial spots number, n_t the observed temporal spots number, $z_{\bar{j}}$ the measure depth, $t_{\bar{n}}$ the measure time, $T(z_{\bar{j}}, t_{\bar{n}}; q)$ the calculated temperature of sea ice at depth $z_{\bar{j}}$ and time $t_{\bar{n}}$ from NNTS, and $T_{mea}(z_{\bar{j}}, t_{\bar{n}})$ the observed data at depth $z_{\bar{j}}$ and time $t_{\bar{n}}$. Thus (30) can be rewritten as

$$\text{(PIPP)}\quad \min \bar{J}(q) = \sum_{\bar{n}=1}^{n_t}\sum_{\bar{j}=1}^{j_z}\big(T(z_{\bar{j}}, t_{\bar{n}}; q) - T_{mea}(z_{\bar{j}}, t_{\bar{n}})\big)^2,$$
$$\text{s.t.}\quad T(z_{\bar{j}}, t_{\bar{n}}; q) \in S(Q_{ad}). \tag{31}$$

From Theorem 4, we can get that there also exists at least one optimal parameter satisfying (31). Next we construct a Hybrid Real Coded Genetic Algorithm to find the optimal solution of (PIPP). The convergence analysis of the algorithm is similar to Ref. [18], so we omit it.

Hybrid Real Coded Genetic Algorithm (HRCGA):

Step 1. Give the size S_p of population; generate the initial population individual $y_i = (y_{1i}, y_{2i}, y_{3i}, y_{4i}, y_{5i}, y_{6i})$, where $y_{ji} \sim \text{rand}(0,1)$, $i \in I_{S_p}$, $j \in I_6$; give the maximal iterative number Max, the given value J_{ter}, the probability P_{HJ} of Hooke–Jeeves search, the accelerated factor α of Hooke–Jeeves search, the descent rate δ of Hooke–Jeeves search; let $q^i = (q_{1i}, q_{2i}, q_{3i}, q_{4i}, q_{5i}, y_{6i})$, $q_{ji} = q_{Lj} + y_{ji}(q_{Uj} - q_{Lj})$, where $i \in I_{S_p}$, $j \in I_6$; set the iterative number $l = 1$.

Step 2. Compute the value $T(z_{\bar{j}}, t_{\bar{n}}; q^i)$ by the semi-implicit finite difference scheme, calculate $\bar{J}(q^i)$ for each q^i, define the fitness value $F_i = 1/\bar{J}(q^i)$, $F_{max} = \max_{i\in I_{S_p}}\{F_i\}$, $F_{ave} = 1/S_p\sum_{i=1}^{S_p} F_i$, the probability $p_i = F_i/\sum_{i=1}^{S_p} F_i$; if $q^i \in Q_{ad}$, then $F_i = 1/\bar{J}(q^i)$, else, $F_i = j_z \cdot n_t$, where $i \in I_{S_p}$.

Step 3. Select S_p individuals from the population y_i according to p_i as the new population individuals $y_i^1 = (y_{1i}^1, y_{2i}^1, y_{3i}^1, y_{4i}^1, y_{5i}^1, y_{6i}^1)$, compute the fitness value F_i^1 of y_i^1, where $i \in I_{S_p}$.

Step 4. Randomly select two individuals from the initial population individual y_i due to the probability p_i and make a random linear combination due to the crossover probability $pc_i = 1/(1 + e^{b_0 \cdot l}) + b_1$ as the population individual $y_i^2 = (y_{1i}^2, y_{2i}^2, y_{3i}^2, y_{4i}^2, y_{5i}^2, y_{6i}^2)$, compute the fitness value F_i^2 of y_i^2, where $i \in I_{S_p}$.

Step 5. Set the mutation probability

$$pm_i = \begin{cases} \frac{d_0}{1+e^{-d_1 \cdot l}}\frac{F_{max}-F_i}{F_{max}-F_{ave}}, & \text{if } F_i \geqslant F_{ave}, \\ p_m, & \text{if } F_i < F_{ave}. \end{cases}$$

For every pm_i, generate a random number $u_{mi} \sim \text{rand}(0,1)$, if $u_{mi} < pm_i$, let $y_{ji}^3 = 1 - p_i$; else, $y_{ji}^3 = y_{ji}$, $i \in I_{S_p}$, $j \in I_6$. The new population individual $y_i^3 = (y_{1i}^3, y_{2i}^3, y_{3i}^3,$

Table 1
The given parameter values of the model

ρ_{pi}	c_{pi}	k_{pi}	S_1	S_2	S_3	S_4	l	z_1	C	$\bar{\alpha}$	Q_s	κ_2
916	2093	2.03	2.2	6	8	13	0.66	0.06	0.9	0.55	62.2	1.5
h_1	h_2	b_0	b_1	c_0	c_1	S_p	Max	P_{HJ}	J_{ter}	α	δ	
1.6666	4.0×10^{-8}	0.05	0.5	2.0	2.0	50	100	0.5	0.1	1	0.5	

Table 2
The identified parameter values from Oct. 20 to Nov. 4, 2006

	γ	β	a_0	a_1	a_2	a_3	e_1	e_2
q^{or}	1.715×10^7	0.1172	1.89	29.19	−77.49	56.38	0.2442	0.06%
q^*	3.0000×10^7	0.4987	3.9998	19.6095	−58.6298	48.0169	0.1708	0.04%
Feasible domain	$[0.5, 3.0] \times 10^7$	[0.001, 0.5]	[1, 4]	[5, 100]	[−100, −1]	[1, 100]	–	–

Table 3
The compare of errors from Oct. 20 to Nov. 4, 2006

z	0.00	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60	0.66
$e_3(q^{or})$	0.0000	0.4140	0.3698	0.3192	0.2595	0.2980	0.1708	0.2908	0.1568	0.0994	0.0368	0.0000
$e_3(q^*)$	0.0000	0.3231	0.2529	0.2030	0.1436	0.2074	0.0942	0.2277	0.1027	0.0656	0.0369	0.0000

$y_{4i}^3, y_{5i}^3, y_{6i}^3$), compute the fitness value F_i^3 of y_i^3, where $i \in I_{S_p}$, $j \in I_6$.

Step 6. Sequence the new individuals y_i^1, y_i^2 and y_i^3 from large to small according to their fitness values F_i^1, F_i^2 and F_i^3; take the front S_p individuals as the new population individual $y_i = (y_{1i}, y_{2i}, y_{3i}, y_{4i}, y_{5i})$, let $q_{ji} = q_{Lj} + y_{ji}(q_{Uj} - q_{Lj})$, $q^i = (q_{1i}, q_{2i}, q_{3i}, q_{4i}, q_{5i}, y_{6i})$, where $i \in I_{S_p}$, $j \in I_6$.

Step 7. Generate a random number $\lambda \sim \text{rand}(0,1)$, if $\lambda < P_{HJ}$, Hooke–Jeeves research for q^1 is carried out and the obtained value is still denoted by q^1, let $q^{S_p} = q^1$, sequence the individuals from large to small according to their fitness values, then goto Step 8; else, goto Step 8.

Step 8. Set $l = l + 1$. If $l \leqslant \text{Max}$ or $\bar{J}(q^1) \geqslant J_{ter}$, goto Step 2; else, take $q^* = q^1$, output q^*.

3.3. An example

In this section, a numerical example illustrates that the physical parameters of NNTS can be identified so as to reduce the bias between the calculated and the measured results. To illustrate the validity of the optimization algorithm and the advantage of the identification method, we take the observed temperature data of Antarctic sea ice at Nella Fjord around Zhongshan Station from Oct. 20 to Nov. 4, 2006 and Jun. 12 to Jun. 25, 2006 measured in the 22nd Chinese Antarctic Research Expedition. There are $j_z = 12$ observed spots distributing averagely from 0 to 0.66 m, and the spatial interval is 0.06 m; there are $n_t = 768$ observed temporal spots, and the sampling interval is 30 minutes. Firstly, we compute the optimal parameter values using the observed temperature data from Oct. 20 to Nov. 4, 2006; then another numerical simulation from Jun. 12 to Jun. 25, 2006 is operated by the optimal parameters which are denoted by q^*; finally, the two results are compared with the numerical results based on the parameters of [3] and [4] which are denoted by q^{or}. The errors are defined by

$$e_1(q) = \sqrt{\frac{1}{j_z n_t}\left(\sum_{\bar{n}=1}^{n_t}\sum_{\bar{j}=1}^{j_z}\bigl(T(z_{\bar{j}}, t_{\bar{n}}; q) - T_{mea}(z_{\bar{j}}, t_{\bar{n}})\bigr)^2\right)},$$

$$e_2(q) = \frac{\sum_{\bar{n}=1}^{n_t}\sum_{\bar{j}=1}^{j_z}|T(z_{\bar{j}}, t_{\bar{n}}; q) - T_{mea}(z_{\bar{j}}, t_{\bar{n}})|}{\sum_{\bar{n}=1}^{n_t}\sum_{\bar{j}=1}^{j_z}|T_{mea}(z_{\bar{j}}, t_{\bar{n}})|},$$

$$e_3(q) = \sqrt{\frac{1}{n_t}\left(\sum_{\bar{n}=1}^{n_t}\bigl(T(z_{\bar{j}}, t_{\bar{n}}; q) - T_{mea}(z_{\bar{j}}, t_{\bar{n}})\bigr)^2\right)}, \quad \bar{j} = 1, \ldots, j_z.$$

The temperature deviation is defined by

$$DT(z_{\bar{j}}; q) = T(z_{\bar{j}}, t_{\bar{n}}; q) - T_{mea}(z_{\bar{j}}, t_{\bar{n}}), \quad \bar{j} = 1, \ldots, j_z,\ \bar{n} = 1, \ldots, n_t.$$

Table 1 presents the given parameter values in the model. Using the optimization algorithm, we obtain the optimal solution q^* as shown in Table 2, where the feasible domains of the identified parameters are presented due to the measured salinity values at Nella Fjord in 2006 and Refs. [3,4,7,8]. Table 3 shows the temperature errors at every observed spot from Oct. 20 to Nov. 4, 2006. Fig. 2 shows the comparison of the temperature deviation for q^{or} and q^* from Oct. 20 to Nov. 4, 2006, where the x-axes express year day from Oct. 20 to Nov. 4, 2006, the y-axes express the ice temperature deviation, the red line denotes the temperature deviation for q^*, the blue line denotes the temperature deviation for q^{or} (for colors see the web version of this article). From Table 2, we can find that the temperature errors for q^* are smaller than those for q^{or}; in Table 3, most values of $e_3(q^*)$ are much smaller than $e_3(q^{or})$. It is illuminated that

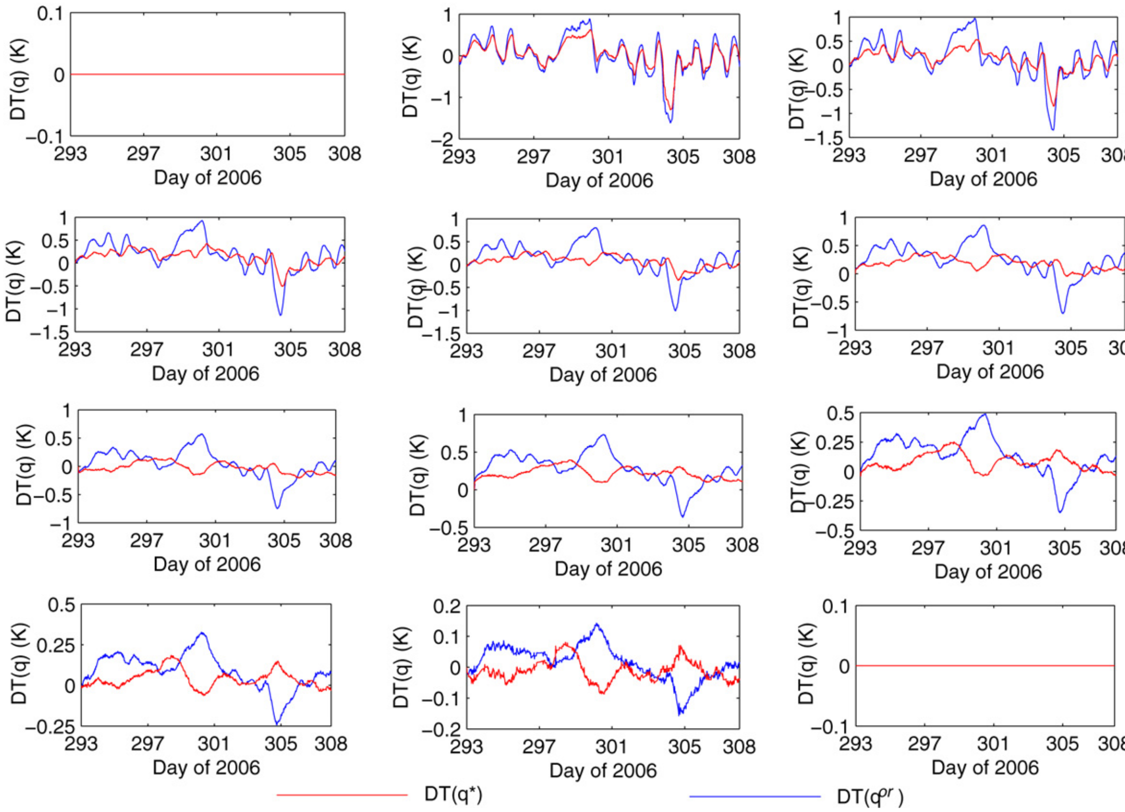

Fig. 2. Comparison of the temperature deviation for optimal parameter q^* and the original parameter q^{or} from Oct. 20 to Nov. 4, 2006.

Table 4
The identified parameter values from Jun. 12 to Jun. 25, 2006

	γ	β	a_0	a_1	a_2	a_3	e_1	e_2
q^{or}	1.715×10^7	0.1172	1.89	29.19	−77.49	56.38	0.5745	0.16%
q^*	3.0000×10^7	0.4987	3.9998	19.6095	−58.6298	48.0169	0.5582	0.16%

Table 5
The compare of errors from Jun. 12 to Jun. 25, 2006

z	0.00	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60	0.66
$e_3(q^{or})$	0.0000	0.7691	0.8004	0.8202	0.8399	0.6675	0.6742	0.4327	0.4018	0.2836	0.1446	0.0000
$e_3(q^*)$	0.0000	0.7396	0.7562	0.7751	0.8030	0.6402	0.6627	0.4364	0.4321	0.3322	0.1943	0.0000

the algorithm is valid. To further illustrate the validity of our method under some conditions, we make another temperature simulation of sea ice at Nella Fjord from Jun. 12 to Jun. 25, 2006 using q^* and compare the results with those for q^{or} as in Tables 4, 5 and Fig. 3. From these tables and figures, we can conclude that the numerical results by our method correspond to the actual characteristics of the sea ice temperature distribution and approach the observed data better than those by q^{or}, thus the algorithm is valid and our method can be used under some conditions.

4. Conclusions

In this paper, we aim to identify the coefficients describing the sea ice salinity and other two parameters in NNTS neglecting the influence of snow and ocean on ice. We have discussed the existence and uniqueness of the weak solution of NNTS, constructed an optimization algorithm solving the parameter identification problem by a new optimization algorithm, and operated another numerical simulation during the Polar Night Time. Compared with the results for q^{or}, the numerical results for q^* indicate that not only the optimization algorithm are valid, but also our model with the optimal parameters can be used, for example, to overcome data gaps.

However, our mathematical framework did not include the impact of snow on sea ice, of the oceanic heat flux and of radiation penetrating through snow on top of the sea ice. To overcome these limitations will be subject to the future work.

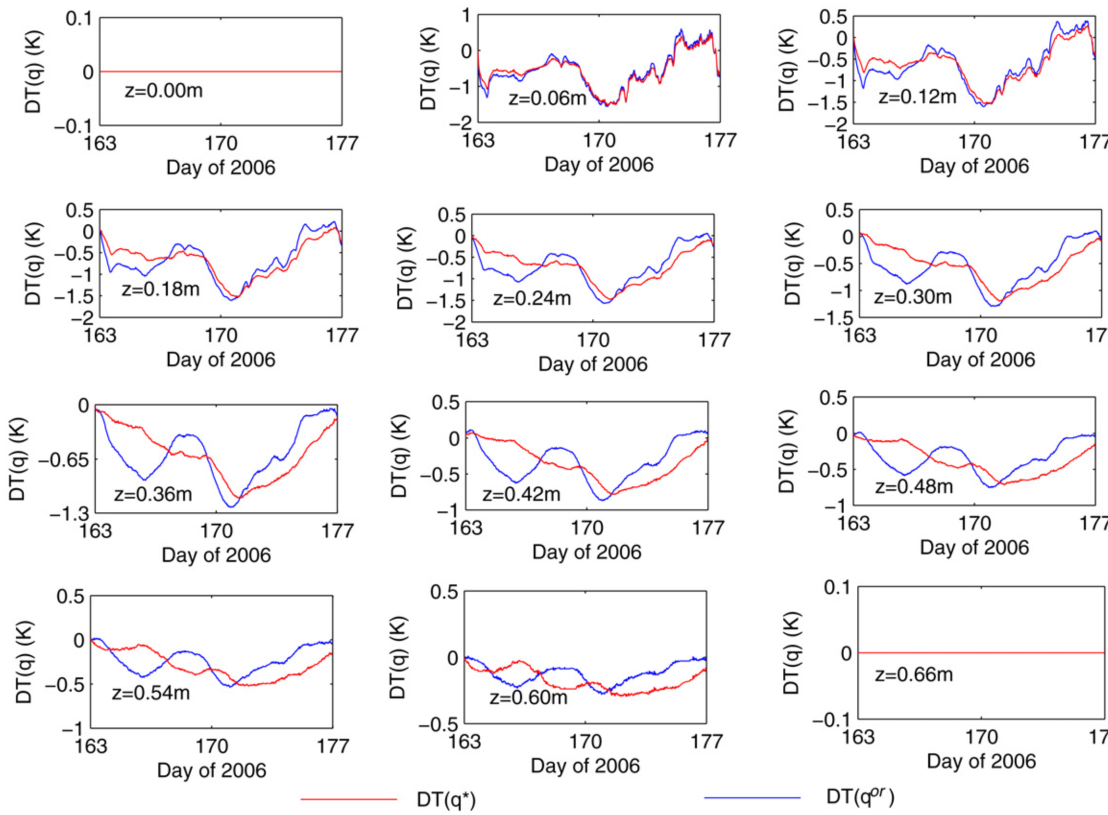

Fig. 3. Comparison of the temperature deviation for optimal parameter q^* and the original parameter q^{or} from Jun. 12 to Jun. 25, 2006.

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